## PS \#4

31. 




| 1 mol monomer | $\mathrm{c}_{0}(\mathrm{mmol})$ | $-\mathrm{COOH}(\mathrm{mmol})$ | $-\mathrm{NH}_{2}(\mathrm{mmol})$ | $\mathrm{N}_{\text {chain }}(\mathrm{mmol})$ | $\mathrm{M}_{\mathrm{n}}{ }^{*} 10^{-3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | - | 5.4 | 4.99 | $(5.4+4.99) / 2$ | 21.7 |
| 2. | V 20.5 | 19.8 | 2.3 | $\left(19.8+\mathrm{c}_{0}+2.3\right) / 2$ | 5.30 |
| 3. | S 10.2 | 21.1 | 2.3 | $(21.1+2.3) / 2$ | 9.66 |
| 4. | H 10.2 | 1.4 | 19.7 | $(1.4+19.7) / 2$ | 10.6 |
| 5. | T 6.7 | 22.0 | 2.5 | $\mathrm{c}_{0}+2.5$ | 12.28 |

* $\mathrm{M}_{\text {caprolactam }}=\mathrm{M}_{0}=113 ; \mathrm{M}_{\mathrm{n}}=\left(1 / \mathrm{N}_{\text {chain }}\right){ }^{*} \mathrm{M}_{0}$
* Reaction 1, 3, 4: The number of chains equals to half of the end groups.
* Reaction 2: Each polymer chain has either an end-capped carbonyl group (here we assume the conversion of acetic acid is $100 \%$ ) or an amine $\left(\mathrm{NH}_{2}\right)$ end group. $\mathrm{N}_{\text {chain }}=(20.5+2.3)(\mathrm{mmol})$ and $\mathrm{M}_{\mathrm{n}}=5 \mathrm{k}$. We can also calculate the average number according to two end groups: $\mathrm{N}_{\text {chain }}=(19.8+(20.5+2.3)) / 2$, and the answer is 5.3 k .
*Reaction 5: Assume all the T is converted into polymer, so there are two kinds of polymer chains: one with a $\mathrm{NH}_{2}$ end group and one with a T unit.

The trend of MW with the additives is consistent with the polymerization mechanism. Water, acid and base play the role as the initiators of ring-opening polymerization. When adding acetic acid, which is a more efficient initiator than water (concentration of initiators increases), so $\mathrm{M}_{\mathrm{n}}$ of the final polymer decreases a lot. When adding diacids or diamines, $\mathrm{M}_{\mathrm{n}}$ doubles as two chains are connected by the difunctional molecule. When adding triacids, it could initiate three chains to form a star polymer with a bigger $M_{n}$.
32.

| Moles water $\left(* 10^{3}\right) /$ mole <br> caprolactam | $\mathrm{M}_{\mathrm{n}}{ }^{*} 10^{-3}$ | $\mathrm{M}_{\mathrm{w}}{ }^{*} 10^{-3}$ | ${ }^{*} \mathrm{M}_{\mathrm{w}} / \mathrm{M}_{\mathrm{n}}=(1+\mathrm{p})$ | p |
| :---: | :---: | :---: | :---: | :---: |
| 49.3 | 13.4 | 20.0 | 1.493 | 0.493 |
| 34.0 | 16.4 | 25.6 | 1.561 | 0.561 |
| 25.6 | 17.9 | 29.8 | 1.665 | 0.665 |
| 20.5 | 19.4 | 36.6 | 1.887 | 0.887 |

* We can't use $\mathrm{M}_{\mathrm{n}}$ to calculate p by using $\mathrm{N}_{\mathrm{n}}=1 /(1-\mathrm{p})$, because of the following reason:

The water-initiated polymerization of caprolactam is characterized by three main equilibrium reactions, which are: ring opening (K1), condensation (K2), and addition (K3). Both ring opening (hydrolysis of caprolactam) and addition (the direct attachment of a molecule of caprolactam to an endgroup of a polymer molecule) are the principal reactions, whereas the condensation equilibrium determines the final degree of polymerization. Before condensation, all the narrow dispersed polymers with $\mathrm{M}_{\mathrm{n}, 0}$ of about several thousand are formed in the solution by chain addition mechanism.

Based on above description, if more water is added, condensation conversion will be less favored, so we will get polymers with a lower molecular weight. Above experimental results proved our expectation.

Calculation of residual water:

| Moles water(*10 <br> caprolactam <br> cale | $\mathrm{M}_{\mathrm{n}}{ }^{*} 10^{-3}$ | $\mathrm{~N}_{\mathrm{n}}$ | $\mathrm{N}_{\text {chain }}(\mathrm{mmol})$ | *Left water(mmol) | $\mathrm{H}_{2} \mathrm{O}$ <br> $\mathrm{wt} \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 49.3 | 13.4 | 118.6 | 8.4 | 40.9 | 0.652 |
| 34.0 | 16.4 | 145.1 | 6.9 | 27.1 | 0.432 |
| 25.6 | 17.9 | 158.4 | 6.3 | 19.3 | 0.307 |
| 20.5 | 19.4 | 171.7 | 5.8 | 14.7 | 0.234 |

[^0]

When conversion is close to 1 , the final water content is about $0.2 \%$ at $225{ }^{\circ} \mathrm{C}$, which is consistent with the value in the book ( $0.15 \%$ @ $290^{\circ} \mathrm{C}$ ).
33.


Above structure illustrate the network formed from $\mathrm{AB}+\mathrm{BB}+\mathrm{A}_{3}$ system
The blue chain is between two branching points.Assume B groups are in excess, the extent of reaction for A groups is p. $r=\frac{v_{A}{ }^{0}}{v_{B}{ }^{0}}<1 ; \mathrm{v}_{\mathrm{A}}{ }^{0}$ is the total A groups

$$
\rho_{A}=\frac{v_{A, f}}{v_{A}{ }^{0}} \quad \rho_{B}=\frac{v_{B(A B)}}{v_{B}{ }^{0}}
$$

There is only one BB unit in the blue chain, so we can calculate the probability of forming such a chain

$$
\mathrm{A}_{3}+\mathrm{BA} \longrightarrow \mathrm{~A}_{2} \mathrm{abA} \quad p \cdot \rho_{B}
$$

$$
\mathrm{A}_{2} \mathrm{abA}+\mathrm{BA} \longrightarrow \mathrm{~A}_{2} \mathrm{ababA} \quad\left(p \cdot \rho_{B}\right)^{2}
$$

Assume m BA unit before BB , the probability is $\left(p \cdot \rho_{B}\right)^{m}$

$$
\begin{gathered}
\mathrm{A}_{2} \mathrm{a}(\mathrm{ba})_{\mathrm{m}-1} \mathrm{bA}+\mathrm{BB} \longrightarrow \mathrm{~A}_{2} \mathrm{a}(\mathrm{ba})_{\mathrm{m}-1} \mathrm{babB} \quad\left(p \cdot \rho_{B}\right)^{m} \cdot p\left(1-\rho_{B}\right) \\
\mathrm{A}_{2} \mathrm{a}(\mathrm{ba})_{\mathrm{m}-1} \mathrm{babB}+\mathrm{AB} \longrightarrow \mathrm{~A}_{2} \mathrm{a}(\mathrm{ba})_{\mathrm{m}-1} \mathrm{babbaB} \quad\left(p \cdot \rho_{B}\right)^{m} \cdot p\left(1-\rho_{B}\right) r p \cdot\left(1-\rho_{A}\right)
\end{gathered}
$$

Assume n AB units after BB , the total probability is $\left(p \cdot \rho_{B}\right)^{m} \cdot p\left(1-\rho_{B}\right)\left[r p \cdot\left(1-\rho_{A}\right)\right]^{n}$ The final unit is $\mathrm{A}_{3}$, so the probability of forming a blue chain is

$$
\left(p \cdot \rho_{B}\right)^{m} \cdot p\left(1-\rho_{B}\right)\left[r p \cdot\left(1-\rho_{A}\right)\right]^{n} \cdot r p \cdot \rho_{A}
$$

Considering different possibilities of m and n , the branch coefficient is expressed as

$$
\begin{gathered}
\alpha=\sum_{m=0}\left(p \cdot \rho_{B}\right)^{m} \cdot p\left(1-\rho_{B}\right) \sum_{n=0}\left[r p \cdot\left(1-\rho_{A}\right)\right]^{n} \cdot r p \cdot \rho_{A} \\
\alpha=\frac{r p^{2} \rho_{A}\left(1-\rho_{B}\right)}{\left(1-p \rho_{B}\right)\left[1-r p\left(1-\rho_{A}\right)\right]}
\end{gathered}
$$

We also know $\alpha=1 /(\mathrm{f}-1)=0.5$, solving p ,

$$
\begin{array}{r}
p^{2}\left(3 r \rho_{A}-2 r \rho_{A} \rho_{B}-r\right)+p\left(\rho_{B}+r-r \rho_{A}\right)-1=0 \\
p_{c}=\frac{r \rho_{A}-r-\rho_{B}+\sqrt{\left(\rho_{B}+r-r \rho_{A}\right)^{2}+4\left(3 r \rho_{A}-2 r \rho_{A} \rho_{B}-r\right)}}{2 r\left(3 \rho_{A}-2 \rho_{A} \rho_{B}-1\right)} \tag{A}
\end{array}
$$

When $\rho_{\mathrm{B}}=1$, we expect that $\mathrm{p}_{\mathrm{c}}>1$, because no network can form between $\mathrm{A}_{3}$ and AB From above expression, we have

$$
\begin{equation*}
p_{c}=\frac{r \rho_{A}-r-1+\sqrt{\left(r-1-r \rho_{A}\right)^{2}}}{2 r\left(\rho_{A}-1\right)}=\frac{1-r \rho_{A}}{r-r \rho_{A}}>1 \tag{B}
\end{equation*}
$$

Expression B derived from A is reasonable, indicating A is right, even though the final expression is not very clear.
34.
(a) Linear: no branch point
(b) Network: two $\mathrm{AB}_{2}$ branch points can be connected by some $\mathrm{A}_{2}$ molecules
(c) Network: two $\mathrm{AB}_{2}$ branch points can be connected by some AB molecules
(d) Network: two $\mathrm{A}_{3}$ points can be connected by both $\mathrm{B}_{2}$ and $\mathrm{A}_{2}$ molecules
(e) Branch: two $B_{3}$ can't be connected together
(f) Network: $\mathrm{A}_{2} \mathrm{~B}_{2}$ can be connected by $\mathrm{A}_{2}$ or $\mathrm{B}_{2}$ molecules
35.

A

aminoanil ine
$\mathrm{M}_{\mathrm{a}}=108$
$\mathrm{Wt} \%=39.31 \%$
$\mathrm{N}_{\mathrm{a}}=50.461$

terephthalic acid
$\mathrm{M}_{\mathrm{t}}=166$
$\mathrm{wt} \%=59.81$
$\mathrm{N}_{\mathrm{t}}=49.951 \quad$ :

B'

benzoic acid
$\mathrm{M}_{\mathrm{b}}=122$ $\mathrm{wt} \%=0.88 \%$
$\mathrm{N}_{\mathrm{b}}=1$

Neglecting the endgroups, $\mathrm{M}_{\mathrm{n}}=24116$; dop $=\mathrm{M}_{\mathrm{n}} /\left(\left(\mathrm{M}_{\mathrm{a}}+\mathrm{M}_{\mathrm{t}}-36\right) / 2\right)=202.42$

$$
\begin{gathered}
r=\frac{v_{A}}{v_{B}+2 v_{B^{\prime}}}=\frac{50.461 * 2}{49.951 * 2+2}=0.9904 \\
N_{n}=\frac{1+r}{1+r-2 r p} \\
202.42=\frac{1+0.9904}{1+0.9904-2 * 0.9904 * p} \\
\mathrm{P}=0.99989
\end{gathered}
$$

If the benzoic acids were doubled

$$
r=\frac{v_{A}}{v_{B}+2 v_{B^{\prime}}}=\frac{50.461 * 2}{49.951 * 2+4}=0.9713
$$

And p was 0.99988 ,

$$
\mathrm{N}_{\mathrm{n}}=68.2
$$

36. 


ph thalic acid
$\mathrm{B}_{2}$

glycerol
$\mathrm{A}_{3}$

ethylene glycol $\mathrm{A}_{2}$

Molar ratio: 1.5
0.5
0.7

$$
\begin{gathered}
p_{c}=\frac{1}{\sqrt{r+r \rho(f-2)}} \\
r=\frac{0.5 * 3+0.7 * 2}{1.5 * 2}=0.9667
\end{gathered}
$$

$$
\begin{gathered}
\rho=1.5 / 2.9=0.5172 \\
p_{c}=0.8257
\end{gathered}
$$

37. 


adipic acid
$\mathrm{M}_{\text {acid }}=146$

hexam et hy lene diamine
$\mathrm{M}_{\mathrm{amine}}=116$

For stoichiometry balanced situation,

$$
N_{n}=\frac{1+r}{1+r-2 r p}
$$

$\mathrm{p}=0.995 \quad \mathrm{~N}_{\mathrm{n}}=10000 /((146+116-36) / 2)=88.5$
$\mathrm{r}=0.9874$
If $\mathrm{p}_{\mathrm{A}}=0.995, \mathrm{p}_{\mathrm{B}}=\mathrm{rp}=0.9824$ (the choice of " A " and " B " is arbitrary)
The end groups are acids or amines, and the ratio is 1.013(0.995:0.9824).
38.

For example: polyurethanes made from 1,4-diisocyanato-benzene and Butane-1,4-diol



[^0]:    *Because each polymer consumes one water molecule, $\mathrm{n}_{\text {water }, \mathrm{f}} \mathrm{f} \mathrm{n}_{\text {water }, \mathrm{i}}-\mathrm{n}_{\text {chain }}$

